

Technical note

A method for measuring joint kinematics designed for accurate registration of kinematic data to models constructed from CT data

K.J. Fischer, T.T. Manson, H.J. Pfaeffle, M.M. Tomaino, S.L-Y. Woo

*Upper Extremity Laboratory, Musculoskeletal Research Center,
Department of Orthopaedic Surgery, University of Pittsburgh, Pittsburgh, PA, USA*

Corresponding Author:

Kenneth J. Fischer, Ph.D.
Department of Mechanical Engineering
University of Kansas
3013 Learned Hall
Lawrence, KS 66045-2234, USA
Tel: (785) 864-2994
Fax: (785) 864-5254
fischer@ukans.edu

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1 **ABSTRACT**

2
3 A method for measuring three-dimensional kinematics that incorporates the direct
4 cross-registration of experimental kinematics with anatomic geometry from Computed
5 Tomography (CT) data has been developed. Plexiglas registration blocks were attached
6 to the bones of interest and the specimen was CT scanned. Computer models of the bone
7 surface were developed from the CT image data. Determination of discrete kinematics
8 was accomplished by digitizing three pre-selected contiguous surfaces of each
9 registration block using a three-dimensional point digitization system. Cross-registration
10 of bone surface models from the CT data was accomplished by identifying the
11 registration block surfaces within the CT images. Kinematics measured during a
12 biomechanical experiment were applied to the computer models of the bone surface.

13 The overall accuracy of the method was shown to be at or below the accuracy of
14 the digitization system used. For this experimental application, the accuracy was better
15 than ± 0.1 mm for position and $\pm 0.1^\circ$ for orientation for linkage digitization and better
16 than ± 0.2 mm and $\pm 0.2^\circ$ for CT digitization..

17 Surface models of the radius and ulna were constructed from CT data, as an
18 example application. Kinematics of the bones were measured for simulated forearm
19 rotation. Screw-displacement axis analysis showed 0.1 mm (proximal) translation of the
20 radius (with respect to the ulna) from supination to neutral (85.2° rotation) and 1.4 mm
21 (proximal) translation from neutral to pronation (65.3° rotation). The motion of the
22 radius with respect to the ulna was displayed using the surface models. This
23 methodology is a useful tool for the measurement and application of rigid-body
24 kinematics to computer models.

1 INTRODUCTION

2 The experimental study of joint kinematics involves the tracking of rigid body
3 motion of bones in three dimensions. To measure this motion, many different
4 methodologies have been developed relying on a diverse array of measuring systems.

5 Stereometric methods (An, *et al.*, 1991; Cappello, *et al.*, 1997; Chao and Morrey,
6 1978; De Lange, *et al.*, 1990a; Kaus, *et al.*, 1997; Spoor and Veldpaus, 1980) use
7 multiple image gathering systems to triangulate the position of markers in the
8 measurement field. These techniques can collect data from a relatively large working
9 space, but accuracy is limited by the number and placement of cameras, the size of the
10 field of view, and the resolution of the images collected. Our preliminary analysis of
11 video motion analysis, for a field of view appropriate for our experiment, was positional
12 accuracy of 1 mm and orientation accuracy of about 1°.

13 Magnetic tracking devices use a magnetic field generator and inductive sensors
14 with three orthogonal coils to determine the position and orientation of the sensor with
15 respect to the generator (An, *et al.*, 1988; Debski, *et al.*, 1995; Hsu, *et al.*, 1996; Jackson,
16 *et al.*, 1994; Milne, *et al.*, 1996). Accuracy of magnetic tracking systems varies with
17 distance from field generator. Again, our preliminary testing indicates these systems can
18 achieve accuracies on the order of 1 mm and 1° for a moderate (1 m³) data collection
19 space.

20 Mechanical linkage systems incorporate six degrees-of-freedom (6 joints) and use
21 potentiometers or optical encoders to measure position of linkage joints and determine
22 the end-to-end position and orientation (Chao, *et al.*, 1980; Gardner, *et al.*, 1996; Hollis,
23 *et al.*, 1991; Kinzel, *et al.*, 1972; McClure, *et al.*, 1998; Townsend, *et al.*, 1977).

1 Mechanical linkages can achieve sub-millimeter accuracy, but the measurement space
2 (linkage reach) tends to be limited to about 50 cm.

3 Most of these techniques utilize a local coordinate system fixed in the body of
4 interest to describe its changing position and orientation with respect to a global reference
5 frame (An, *et al.*, 1979; Chao, *et al.*, 1980; De Lange, *et al.*, 1990b; Debski, *et al.*, 1995;
6 Engin, *et al.*, 1984; Kinzel, *et al.*, 1972). However, none of the methods described above
7 have an intrinsic means of relating the object geometry (anatomy) with the local
8 coordinate system. Often, to determine these relationships, manual point digitization of
9 anatomical landmarks is performed. This technique is subject to errors with respect to
10 the accuracy and repeatability of the manual digitization of the landmarks. Thus, an
11 improved registration system for the local coordinate system could help improve the
12 accuracy of previously described kinematic data collection methods.

13 In order to use the rigid body kinematics (for instance, ligament insertion site
14 motions) to accurately estimate soft tissue elongation, more accurate and precise means
15 of measuring the kinematics are needed. The objectives of this study were to develop
16 methodologies with improved accuracy for: 1) relating the anatomical geometry of the
17 specimen, determined from medical imaging datasets, with an accurate local coordinate
18 system during experimentation; 2) determining the current position of orientation of the
19 local coordinate system during an experiment (and thus, allowing measurement of
20 discrete kinematics of anatomical specimens). The proposed method is unique in the use
21 of registration blocks to locate the local coordinate system within imaging data sets and
22 within an experimental testing space, with accuracies on the order of 0.1 mm and 0.1°.

23

METHODS

Registration Block Coordinate Systems

The position and orientation of a local coordinate system in a global reference frame (**Figure 1**) can be entirely described by the global position of the local coordinate system origin and the global description of the three orthogonal unit vectors (**i, j, k**) that represent the axes of the local coordinate system (Kinzel, *et al.*, 1972). The registration block local coordinate system is based on a corner of the block as the origin and the three block edges as the axes. Points from three pre-selected contiguous faces of the registration blocks were used to define local coordinate systems relative to the global measurement system. The two global measurements systems considered here are the computed tomography (CT) reference frame, in which the blocks are viewed in the medical image data set, and the Microscribe® reference frame, in which sides of the blocks were digitized with the Microscribe-3DX spatial digitizer (Immersion Corporation, San Jose, CA, USA). Either measurement system yields many points from the three surfaces of the registration block.

Using a least squares optimization, approximately 100-150 points per face were fit to the plane equation,

$$Ax + By + Cz + D = 0, \quad (1)$$

where the parameters A, B, and C represent the global X, Y, and Z components of a vector normal to the plane. The optimization minimized the global sum, Φ , of the distances, d_i , from each digitized point to the plane by adjusting the plane equation parameters, A, B, C and D according to the equation

$$\text{Min}[\Phi(A, B, C, D)] \quad \Phi = \sum_{i=1}^N d_i \quad d_i = \frac{|Ax_i + By_i + Cz_i - D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$1 \quad , \text{ where } \quad , \text{ and } \quad , \quad (2)$$

2
3 where x_i , y_i , and z_i correspond to the global coordinates (from CT or Microscribe®) of
4 each digitized point.

5 The three planes resulting from the optimization described above were used to
6 form a local coordinate system on the registration block. The intersection of the three
7 planes represented the vector origin, \mathbf{O} , of the local coordinate system in the global
8 reference frame. The intersection was computed from the plane equations using
9 Gaussian elimination. The outward facing normal unit vector for each plane, \mathbf{n}_i , was
10 derived from the parameters (A, B, and C) from each plane equation (**Figure 2**). To
11 assure an orthogonal coordinate system, the normal vectors were not directly assigned as
12 coordinate system axes. The unit vector, \mathbf{i} , was taken directly as the unit vector \mathbf{n}_1 :

$$13 \quad \mathbf{i} = \mathbf{n}_1 \quad (3)$$

14 The unit vector \mathbf{k} was calculated as the cross product of the unit vector \mathbf{i} with the normal
15 to plane 2:

$$16 \quad \mathbf{k} = \mathbf{i} \times \mathbf{n}_2 \quad (4)$$

17 The cross product of \mathbf{k} and \mathbf{i} yielded the unit vector \mathbf{j} :

$$18 \quad \mathbf{j} = \mathbf{k} \times \mathbf{i} \quad (5)$$

19 The result was an orthogonal right-handed local coordinate system with origin at
20 position \mathbf{O} and unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} relative to the global reference frame (**Figure 2**).

21 An affine transformation matrix $\mathbf{T}_{g \leftarrow l}$ that transforms a position vector from the
22 local block coordinate system to the global reference frame was then formed from \mathbf{O} and
23 \mathbf{i} , \mathbf{j} , \mathbf{k} as:

$$\mathbf{T}_{g \leftarrow l} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ O_X & i_X & j_X & k_X \\ O_Y & i_Y & j_Y & k_Y \\ O_Z & i_Z & j_Z & k_Z \end{bmatrix} \quad (6)$$

This assembly of the transformation matrix is well established in the kinematics literature (Paul, 1981). The inverse transformation allows points (geometry) in the global reference frame to be transformed to the local coordinate system.

Accuracy of Coordinate Systems

To assess the accuracy of the method, the relative position and orientation was calculated between the two far corners of a machinist's 1-2-3 gage block (1x2x3 inches, Enco Corp., Cincinnati, Ohio, USA) machined to tolerances within ± 0.003 mm and $\pm 0.0005^\circ$. In the experiment, the Microscribe-3DX® (with a stated accuracy of ± 0.2 mm) was used to digitize the three faces forming each of two far corners of the machinist's block. The two local coordinate systems and the transformation between them were calculated using the method outlined above. The difference between the specified block dimensions and the calculated distance between the local coordinate systems was used to determine the positional accuracy of the registration block method with the Microscribe. In addition, any deviation from 90° rotations between the coordinate systems was considered angular error due to the Microscribe coordinate system determination.

A similar assessment of accuracy was conducted for locating coordinate systems from medical image data collected with a clinical CT scanner (GE Genesis Highspeed Advantages 9800, General Electric Medical Systems, Waukesha, WI, USA). In this case, Plexiglas blocks were machined to tolerances of ± 0.01 mm, and were used in a similar

1 fashion to the machinists gage block in the previous assessment. Images of the Plexiglas
2 blocks were obtained from the CT scanner, with a voxel size of 0.2 mm x 0.2 mm x 1mm.
3 Points from three adjacent sides of the block were collected from the image data to locate
4 each of two coordinate systems from far corners of the Plexiglas block. Again, the
5 differences between block dimensions and the locations of coordinate systems
6 determined from CT data for the far corners of the block were used to evaluate positional
7 accuracy, and variations from 90° rotations were considered to be angular error in the
8 coordinate system determination from CT images.

9 10 *Application to Forearm Rotation*

11 A standardized protocol approved by the institutional review board was followed,
12 with respect to use of human tissues. A human cadaveric forearm was dissected free of
13 muscle and soft tissue in the mid-forearm leaving ligamentous structures intact. Precisely
14 machined orthogonal Plexiglas registration blocks were rigidly cemented to the radius
15 and ulna in mid-forearm (**Figure 3**). The forearm was scanned under a standard CT bone
16 protocol (80 kvp, 140 ma, 1 s), at 1 mm intervals and 1 mm slice thickness using a field
17 of view of 10 cm. The image data was used to construct 3-D computer models of the
18 bones and to determine the registration block local coordinate system within the global
19 CT reference frame.

20 Image analysis software developed in-house was used to segment and digitize
21 point contours of the bone surfaces, and the faces of the registration blocks. The contours
22 corresponding to the external surface of the radius and the ulna were first segmented from
23 the image set. The resulting contours were connected with triangles to form computer

1 models using the freeware package Nuages© (Geiger, 1993). In addition, the in-house
2 image analysis software was used to segment the registration blocks. Then edge points
3 corresponding to three pre-selected, contiguous faces of the registration blocks were
4 collected, to form the local coordinate system. The position vectors of bone surface
5 points on the radius, \mathbf{r}_{ct} , and ulna, \mathbf{u}_{ct} , in the global CT coordinate system, were then
6 expressed in their respective local coordinate systems via the global to local
7 transformation,

$$8 \quad \mathbf{r}_r = \mathbf{T}_{r \leftarrow ct} \bullet \mathbf{r}_{ct}, \quad (7)$$

9 where \mathbf{r}_r is the position vector of the point on the radius with respect to the local radius
10 (registration block) coordinate system. This operation was repeated to express all of the
11 vectors for points on the radius in terms of the local radius coordinate system and all of
12 the vectors for points on the ulna in terms of the local ulna coordinate system. All
13 calculations were performed using Mathematica® (Wolfram Research, Champaign, IL,
14 USA.

15 The forearm was then subjected to an experiment in which the position and
16 orientation of the forearm bones were measured in different positions of forearm rotation
17 (Pfaeffle, *et al.*, 1999). The forearm was placed in supination, neutral rotation, and
18 pronation. In each position, 100-200 points from each of the three pre-selected
19 contiguous faces of the registration blocks (the same faces used for the CT data) were
20 digitized using the Microscribe-3DX® spatial digitizer. From these points, the local
21 coordinate systems for the radius and ulna were calculated relative to the global
22 Microscribe® coordinate system, using the method described above. The transformation
23 from the local radius coordinate system to the global Microscribe® coordinate system,

1 $T_{m \leftarrow r}$, was also assembled as described above. For each forearm rotation position, all of
2 the vectors for points on the radius and all of the vectors for points on the ulna were
3 transformed from the local coordinate systems to the global Microscribe® coordinate
4 system,

$$5 \quad \mathbf{r}_m = T_{m \leftarrow r} \bullet \mathbf{r}_r, \quad (11)$$

6 where \mathbf{r}_m is the position vector of the point of interest on the radius with respect to the
7 global Microscribe® coordinate system (**Figure 1**). By expressing bone geometry
8 relative to the Microscribe® coordinate system, position of the bones measured during
9 the experiment with the Microscribe® were visualized with the computer models.

10 To describe the motion of the radius from supination to neutral rotation and from
11 neutral rotation to pronation, kinematic transformations were calculated for the motion of
12 the radius with respect to the ulna. The kinematic transformation describing the motion
13 from supination to neutral rotation was then calculated as the position and orientation of
14 the local radius coordinate system in neutral rotation with respect to the local radius
15 coordinate system in supination, $T_{s \leftarrow n}$. This procedure was repeated to obtain the
16 transformation for the motion of the radius from neutral rotation to pronation, $T_{n \leftarrow p}$. The
17 transformation for each of these two motions was decomposed into a single rotation, ϕ ,
18 about and translation, s , along a screw-displacement axis using the techniques of Kinzel
19 et al. (1972).

20 The surface models of the bones constructed from the CT data were used to
21 display the motions of the radius with respect to the ulna, measured from the experiment.
22 The computer models were rendered in Tecplot® (Amtec Engineering, Seattle, WA,
23 USA) for the positions of supination, neutral rotation, and pronation.

RESULTS

The unique registration block method achieved the goal of accurately relating the specimen geometry from medical imaging to the experimental positions. In addition, the registration block method achieved the goal of improved accuracy. The accuracy study revealed that position and orientation accuracy using the Microscribe® digitizer were better than ± 0.1 mm for position and better than $\pm 0.1^\circ$ for orientation. The accuracy using the CT scan data was better than ± 0.2 mm and better than $\pm 0.2^\circ$, respectively. For the spatial digitizing system, the position and orientation accuracy for the registration block method was found to be better than the stated accuracy of the digitizer. The accuracy with the CT data was essentially equal to the in-plane resolution of the images, and substantially better than slice thickness dimension.

The registration blocks were also demonstrated to allow measurement of discrete kinematics. Our analysis of forearm rotation, using the screw-displacement axis, showed 0.1 mm (proximal) translation of the radius (with respect to the ulna) during forearm rotation from supination to neutral. The rotation about the screw axis from supination to neutral forearm rotation was 85.2° . For the rotation from neutral forearm rotation to pronation, a 1.4 mm (proximal) translation of the radius was found. The rotation about the screw axis from neutral forearm rotation to pronation was 65.3° . Actual experimental kinematics were be visualized using the surface models of the bones develop from the CT data (**Figure 4**).

1 **DISCUSSION**

2 A unique methodology has been developed for determining the three-dimensional
3 position and orientation of rigid bodies. The primary advantage of this method is direct
4 and *accurate* registration of data from different measurement systems, such as CT data
5 and spatial digitizers. The result is that kinematic measurements from biomechanical
6 experiments can be easily applied to computer models of the bones. The methodology is
7 highly accurate, and (presumably through the least squares optimization for each
8 registration block plane) actually improves upon the accuracy of the base measurement
9 system. Thus, this methodology could substantially improve the accuracy of locating
10 local coordinate systems, relative to techniques such as manually digitizing anatomical
11 landmarks.

12 While this method presented may improve accuracy, it also has some limitations.
13 To achieve the accuracy, the registration blocks must be rigidly attached to the object of
14 interest, to assure the rigid-body assumption holds. Thus, the method may not
15 significantly improve the accuracy of in vivo kinematics, where the blocks would not be
16 rigidly attached to the bones. The method could, however, still provide a direct link
17 between the bone geometry (from medical imaging) and the kinematics, if the blocks
18 were attached to the skin (or brace) and imaging was performed just prior to kinematic
19 testing. Another drawback of this methodology as currently implemented is that it does
20 not intrinsically allow for capture of data while the bodies are in motion. A simple
21 extension of the registration block methodology described, however, will allow the
22 specimen geometry to be accurately registered with any dynamic measurement system,
23 such as instrumented spatial linkages (Kirstukas, *et al.*, 1992a; Kirstukas, *et al.*, 1992b).

1 To achieve registration between the blocks (the anatomical geometry) and the dynamic
2 measurement system, the body in question must be fixed in one (arbitrary) position.
3 While the specimen position remains unchanged, the registration block faces would be
4 scribed and position and orientation data would be collected with the dynamic
5 measurement system. To obtain the data in a common coordinate system, the most
6 straightforward method is to use a (calibrated) digitizing stylus attached to the free end of
7 the dynamic measurement system (prior to mounting the dynamic system to the body and
8 collecting data in the fixed position). If this is not possible, a separate digitizing system,
9 such as the Microscribe®, could be used to digitize the block and the base of the dynamic
10 measurement system. In either case, reducing the data from both measurement types
11 would provide the direct (fixed) transformation from the dynamic system measurements
12 to the position and orientation of the registration blocks (and the bone geometry). Thus,
13 the position and orientation of the bone could be calculated for any arbitrary (or transient)
14 position of the dynamic measurement system. While the accuracy of the dynamic
15 positions and orientations would be limited by the dynamic measurement system used,
16 the accuracy of geometry registration to the dynamic measurement system would be as
17 reported above.

18 While demonstrated in this study for a specific application, the registration block
19 digitization method can easily be generalized as described above. The method could
20 allow for the registration of kinematic data from multiple systems for the same body,
21 simply by collecting data from each system with the body in a fixed position. For
22 instance, a goniometer may measure the precise motion at a joint, and a magnetic
23 tracking system may measure the global motion of a limb. The registration block

1 technique would align both dynamic measurement systems with the geometry of the
2 bone. The registration block scribing technique can also be used to align data from a
3 force sensor with anatomical geometry, or to align multiple force sensors for combining
4 and/or analyzing the data (Pfaeffle, *et al.*, 1999).

5 Because the registration block technique is tailored to 3-D computer models of the
6 bones, it can be used for accurate computer visualization of experimental kinematics (as
7 illustrated). We have begun working on the display real-time joint kinematics by
8 imaging the specimen and reducing the data in advance of testing, and by coupling
9 registration block technique with a dynamic measurement system. Thus, during a
10 biomechanical experiment, though the soft tissue around the joint has been kept intact,
11 we hope to be able to display the motion of the bones at the joint. In addition,
12 adaptations of these methods could be employed for computer-aided surgery and pre-
13 operative planning scenarios, in which very accurate registration between the radiological
14 study and the operating theater is crucial. By providing a link between anatomy, and
15 kinematics, this method presents a useful tool for biomechanical experiments and
16 analyses.

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22
23 .

NOMENCLATURE

X, Y, Z	Axes of global reference frame
\mathbf{O}	Vector origin of the local coordinate system with respect to the global reference frame
\mathbf{o}	Vector origin of the local coordinate system with respect to the local reference frame
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Orthogonal unit vector axes of local coordinate system
A, B, C, D	Parameters of the plane equation
d_i	Distance from a digitized point to a plane
$\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$	Normal vectors to planes 1, 2, 3 formed by the three faces of the cube
$\mathbf{T}_{g \leftarrow l}$	Transformation matrix from the local coordinate system to the global reference frame
\mathbf{r}_{ct}	Representative position vector of a surface point on the radius with respect to the global CT reference frame
$\mathbf{T}_{r \leftarrow ct}$	Transformation from the global CT reference frame to the local radius block coordinate system
\mathbf{r}_r	Representative position vector of a surface point on the radius with respect to the local radius block coordinate system
$\mathbf{T}_{m \leftarrow r}$	Transformation from the local radius block coordinate system to the global Microscribe reference frame
\mathbf{r}_m	Representative position vector of a surface point on the radius with respect to the global Microscribe reference frame
$\mathbf{T}_{n \leftarrow s}$	Displacement matrix describing the motion of the radius from supination to neutral rotation
ϕ	Rotation about a screw displacement axis
s	Translation along a screw displacement axis

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Figure Legends

Figure 1. A local coordinate system ($\mathbf{x}, \mathbf{y}, \mathbf{z}$) fixed in a rigid body (for this example, the radius) is defined with respect to the global reference frame ($\mathbf{X}, \mathbf{Y}, \mathbf{Z}$) by the position vector, \mathbf{O} , and a direction cosine matrix (assembled from the local unit vectors, $\mathbf{i}, \mathbf{j}, \mathbf{k}$). An example point on the radius is shown with respect to the local radius coordinate system (\mathbf{r}_r , with respect to $\mathbf{i}, \mathbf{j}, \mathbf{k}$) and with respect to the global (for this example the Microscribe) coordinate system (\mathbf{R}_M , with respect to $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$). The transformation of the point in the local coordinate system to the global system is defined as $\mathbf{R}_m = \mathbf{T}_{m \leftarrow r} \bullet \mathbf{r}_r$.

Figure 2. On the left, a registration block cube with discrete points digitized on each of three of its faces. On the right, the discrete points for each face have been fitted to three planes and a local coordinate system ($\mathbf{i}, \mathbf{j}, \mathbf{k}$) has been formed from the vectors $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ normal to each plane.

Figure 3. Close-up view of the forearm in the experiment, showing the registration blocks attached to each bone. The arrows on the registration blocks indicate the direction to scribing to produce an outward normal by right-hand-rule.

Figure 4. Radius and ulna geometry constructed from CT data and displayed in positions determined from kinematic data from the forearm experiment. The small proximal translations (0.1 mm from A to B, and 1.4 mm from B to C) of the radius with respect to the ulna (stationary in these images) are not readily apparent. However, the rotation of the radius about the ulna (85.2° from A to B, and 65.3° from B to C) is very evident.